Maths problems and solutions

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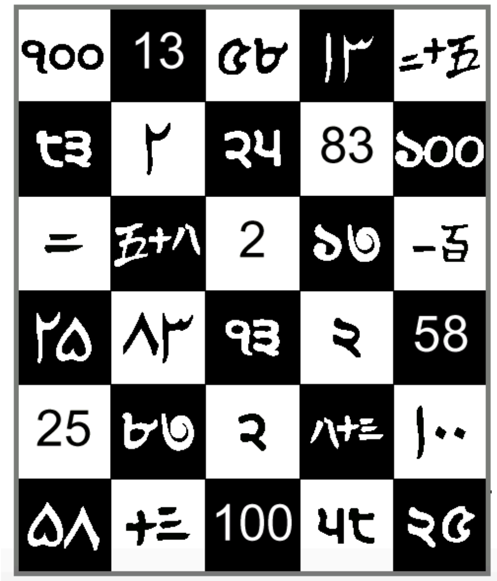
In the last three issues of The Teacher we have provided what we are hope have been some fun, stimulating mathematics problems to solve. On the 23 March a group of Joburg-based teachers got together with us at Wits university to discuss the different ways we thought about the problems, to look at our solutions and – in some cases – to work with others to get the solutions to the problems that had, in the words of one teacher, "given me sleepless nights!". In this article, we share some important ideas that emerged from our discussions and, of course, the solutions to the problems.

Question 1: There are six numbers written in five different scripts in the grid below. Can you sort out which is which?



It was fun to try and figure out what the various symbols might mean just by looking at their shapes. Assuming that each script used some form of place value system we decided that

२ ২ = ٢

probably all represented 2.

However, we realized that if we wanted to solve the problem fully we needed to work in a systematic way. So, we drew up a table that contained the numbers in the script we recognised in column A, and the different versions of two alongside:

A	B	C	D	E
2	٢	२	২	=
13				
25				
58				
83				

100				

Knowing the 2s, we knew these icons should appear as in the tens position of each script's version of 25. For example, if we started with the script where ٢ is 2, we could see from the list in column A that we should have a 25 that starts with ٢ . Looking through the grid we found ٢٥. This led us to conclude that the "upside down heart" is 5.

Knowing the 5 led us to look next for a 58 containing the 'upside down heart' in the tens position. We found ٥٨. From this we concluded the "upside down V" is 8 and then that ٨٣ is 83. As we knew what 3 looks like, we could see that ١٣ is 13. And finally knowing what 1 looked like, we figured out that ١٠٠ must be 100, completing column B.

Using the same process, try filling the other columns yourself.

The script that was slightly different was the one that started with 二 . This suggested that 三 was likely to be '3' as well. Looking for 13, which contains three units (and therefore should contain 三) and '25' (which should contain 二 in the tens position). We found 十三 and 二十五. Both of these icons contained the 十 . This led us to look at the structure of these two numbers:
13 is made up of 1 ten and 3
25 is made up of 2 tens and 5
This pointed to 十 standing for '10'.

These are the Chinese numerals. We hope this explanation helps you complete the matching for this script as well.

Question 2:

In the multiplication below, some of the digits have been replaced by letters and others by asterisks. Where a digit has been replaced by a letter, the same letter represents that same digit and different letters represent different digits. (e.g. if A = 1 in the first line, then A = 1 in the second line. And A and B must be different digits). Asterisks can stand for any digit and can be the same or different and can also be the same as A, B or C.
Can you reconstruct the original multiplication?

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      A B C
      B A C
      -----
    * * * *
    * * A
* * * B
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* * * * * *
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Many of us found this original question tough. So we started with a simpler question:

Use each of the digits 0; 1; 2; 3; 4; 5; 6 **exactly once** to fill in the gaps to make the number sentences true:

☐	+	5	=	☐
☐	×	☐	=	☐
5	-	☐	=	☐

For this one many of us used trial and error – just trying different until we found something that worked. But in the process many of us realized that we could have started by figuring out that the middle line had to be 2 × 3 = 6 (or 3 × 2 = 6) as that is the only multiplication using this digit range that doesn't repeat digits. Crossing out the numbers used leaves 0, 1, 4 and 5. This means the only option for line 1 is 0 + 5 =5, leaving us with 4 and 1 which meant the last line could be 5 – 1 = 4 or 5 – 4 = 1.

Returning to the original problem, we decided to work systematically through the clues: Looking at the first line of stars we could see that C × "ABC" must be a four digit number, so C could not be 1. We then needed to check whether C was 2, 3, 4, 5, 6, 7, 8 or 9. As this left us with a lot of options to consider for C, we moved on to look at what we could deduce from the second line of stars.

Looking at the second line of stars we could see that "ABC" x A must be a 3-digit number. This only happens if A is 1, 2 or 3. We also know the product, A × C, must end with the digit A. So we worked through whether this could happen for A = 1, 2 or 3 (keeping in mind that we know C can't be 1).

- We can't have A =1, since 1 × C = C and A and C can't be the same number.
- It is possible to have A = 2 as long as C = 6, since 2 × 6 = 12

- We can't have A = 3 because if we look at the first 9 multiples of 3 (3; 6; 9; 12; 15; 18; 21; 24; 27) none of them end in 3 except the first one – and we know C can't be 1.

We then had A = 2 and C = 6 and we just needed to figure out the value of B.
Moving to the last line of stars, we deduced that for "ABC" x B to be a 4-digit number, B had to be 4 or bigger since "ABC" was a 'two hundred and something' number. We also needed the B x C i.e. B × 6 product to end with the B digit. There are only two options for this: B=4 or B=8. But ABC = 246 would not give a 4-digit number in the last line of stars, so B must =8.
So the problem must be 286 x 826.

Question 3:
We did not get a chance to discuss question 3 at the Wits Maths Circle, but offer an explanation here. We'd welcome any better, clearer explanations!

1. Take any three-digit number where the first and last digits differ by 2 or more.
2. Reverse the digits, and subtract the smaller from the larger one.
3. Add to this result the number produced by reversing its digits.
   What do you find? Is there a way of explaining what you find?

Try out a few examples e.g.
623 – 326 = 297 and 297 + 792 = 1089
751 – 157 = 594 and 594 + 495 = 1089.
We soon see that the answer is always 1089.

Why? Well if we take "abc" – "cba" what we are actually saying is
(100a + 10b + c) – (100c + 10b + a) = 99a – 99c = 99(a-c).
This means that after the subtraction we have a multiple of 99.
i.e. we have 198 or 297 or 396 or 495 or 594 or 693 or 792.
And we can see for each of these the hundreds and units digit add to 9 and the tens unit is 9. So when we reverse the order of the digits and add them we'll get 1089.

<u>Start a Wits Maths Circle at your school!</u>

Below is the first of the next set of monthly maths problem for you to think about and try. You can try it on your own, with colleagues in your school, with your learners or with your own children. Or all of these! You can email your solutions to us and/or bring them along to a Wits Maths Circles event for primary teachers at the **Wits School of Education** that we will hold towards the end of term 2 at Wits School of Education. Our aim is to discuss different participants' ways of thinking about the problems that we set – it will be a relaxed environment for all of us to share and discuss our different approaches.

Let us know if you want to come to a Wits Maths Circle event on this address:
primary.maths@wits.ac.za. And you can email solutions to us through this address as well – write 'Wits Maths Circles-April problem' in the Subject line. Solutions and different teachers' ways of thinking about the first three months' problems will appear in the July issue.

Wits Maths Circles is an initiative focused on primary mathematics teacher development through building platforms and spaces for primary teachers to work on mathematics – for themselves and for their teaching – in fun, supportive and non-threatening ways. It is an initiative built on a partnership between the Wits Maths Connect – Primary project at Wits and The Teacher. The 'ticket' for entry to a Wits Maths Circle event is some work on one or more of the problems that have been set in The Teacher during that term.

Wits Maths Circles – April Problem

A chess board is made up of 8 rows of 8 squares. Using the lines on the chessboard
a) How many squares of any size can you make?
b) How many rectangles of any size can you make?